**UNIVERSITY OF TORONTO  
Faculty of Arts and Science**

**DECEMBER 2015 EXAMINATIONS**

**PHL245H1F**

**Alex Koo**

**Duration - 3 hours**

**Examination Aid: Sheet with rules (provided)**

Last Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

First Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Student Number: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Answer **ALL** questions on the exam paper.

Use examination booklets for rough work if needed.

If you need further space, use an examination booklet and clearly indicate on the exam paper where your solution is.

The exam consists of 14 pages. Pages 2-12 have questions on them.

The final two pages 13-14 are an aid sheet and may be detached from the rest of the exam.

Part I: Semantics (30 marks)

1. Provide a shortened truth-table that demonstrates the following sentences is not a tautology. (3)

(T∨S→~P)∨(~(P∨Q)↔R∧~S)

2. Consider the following argument where φ, ψ, and π are sentences in predicate logic:

φ is a logical truth.

φ is logically equivalent to ψ.

{ψ, π} is consistent.

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∴ π is a logical truth.

Is the argument valid? Why or why not. Explain your answer in reference to the actual argument. (4)

3. Consider φ1, φ2, …, φ10 where each individual φk is a sentence in predicate logic. Suppose there is an interpretation which makes all of φ1, φ2, …, φ10 true, as well as an additional sentence, ψ.

a. Is the argument φ1. φ2. … φ10. ∴ ψ valid? Explain your answer. (2)

b. Consider the set Γ: { φ1, φ2, …, φ10, ψ}. Are they any properties that we can conclude about Γ? Explain your answer. (2)

4. Provide an English explanation that shows the following sentence is a tautology/logical truth. (3)

∀x∃y(F(xy)∧Gy)→∃x∃y(Gx∧F(xy))

5. Provide intensional (English language) interpretations to demonstrate that the following sentence is contingent (3)

∃x∀y(Fx∧(Hy→L(xy)))→∀x(Fx→Hx)

6. Provide an extensional interpretation (finite abstract model) of the following set of sentences that shows them to be consistent. (3)

{ ∃x(Fx∧Gx), ∀x(Fx→∃y(Gy∧~Hy∧Hx)), ∃x~(Hx↔Fx) }

7. Provide an extensional interpretation (finite abstract model) of the following argument that shows it to be invalid. (3)

∃x(Fx∧∀y(Hy→G(xy)). ∀x(Hx∨Ax). ∀x(Fx→∃y(Ay→G(yx)). ∴ ∀x(Hx→G(xx))

8. Briefly explain why we cannot use truth-tables to semantically analyze predicate logic sentences. (2)

9. Consider the following argument

∃x∀y(Bx∧C(xy)). ∀x(Ax→∃y~C(xy)). ∴ ∀x(Ax→~C(xx))

a. Provide a truth-functional expansion to a universe of discourse with 2 elements (4)

b. Provide an extensional interpretation (finite abstract model) that shows the argument to be invalid. (1)

Part II: Symbolization (34 Marks)

Symbolize questions 1-9, and translate question 10 using the abbreviation schemes.

1. Albert’s best friend doesn’t like cheesecake and apple sauce. (3)

a0: Albert. b1: The best friend of *a*. A1: *a* is apple sauce. C1: *a* is cheesecake. ‘

L2: *a* likes *b*.

2. Not all singers worship Taylor Swift, who is the most popular singer on Earth. (4)

a0: Taylor Swift. b0: Earth. A1: *a* is a singer. B2: *a* worships *b*. C2: *a* is on *b*.   
D2: *a* is more popular than *b*.

3. Some people like beer unless it is hoppy, however some other people only like wine. (4) H1: *a* is a person. B1: *a* is beer. C1: *a* is wine. F1: *a* is hoppy. D2: *a* likes *b*.

4. Jenny’s cousin is Martha’s boss’ yoga instructor, and she isn’t nice. (3)

a0: Jenny. b0: Martha. b1: The boss of *a*. c1: The cousin of *a*.   
d1: The yoga instructor of *a*. N1: *a* is nice.

5. Among athletes, those who eat some sort of healthy food every day will sign at least one big deal. (3)

A1: *a* is an athlete. D1: *a* is a day. F1: *a* is healthy food. E1: *a* is a big deal.   
G2: *a* signs *b*. H3: *a* eats *b* on *c*.

6. The Blue Jays are the only team that the person who lives between Marlowe and Wren watches. (4)

b0: The Blue Jays. c0: Marlowe. d0: Wren. b2: The person who lives between *a* and *b*. A1: *a* is a team. D2: *a* watches *b*.

7. Although at most one dog doesn’t hate Alex, all cats do. (3)

a0: Alex. C1: *a* is a cat. D1: *a* is a dog. E2: *a* hates *b*.

8. Good students either work hard or are brilliant, and in the former case they visit some library or another every day. (4)

A1: *a* is a student. B1: *a* is brilliant. G1: *a* is good. H1: *a* works hard. D1: *a* is a day.   
L1: *a* is a library. M3: *a* visits *b* on *c*.

9. Symbolize the following ambiguous sentence 2 different ways. Provide an English sentence that clarifies the meaning of each of your symbolizations. (3)

Someone is renting a car every day.

F1: *a* is a person. C1: *a* is a car. D1: *a* is a day. A3: *a* rents *b* on *c*.

10. Translate the following sentence into IDIOMATIC English using the following abbreviation scheme. (3)

∃x(Fx∧Gx∧∃y(Fy∧Gy∧~x=y∧∃z(Az∧D(xz)∧D(yz)∧∀w(Aw∧D(xw)∧D(yw)→w=z))))

F1: *a* is young. G1: *a* is a kid. A1: *a* is a soccer ball. D2: *a* kicks *b*.

Part III: Derivations (36 marks)

1. Prove that the following argument is valid using a derivation. You may only use the **10 basic rules of sentential logic and the 3 basic rules of predicate logic**. (9 marks)

∃x(Fx∨Hx)→∃x∀yG(xy). ~∃w(~Fw∧∃zG(zw)). ∴ ∀x(Fx↔∃yG(yx)).

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2. Prove that the following argument is valid using a derivation. You may only use the **10 basic rules of sentential logic and the 3 basic rules of predicate logic**. (9 marks)

~∃x(Fx∧∃yG(xy)). ~∀z(Cz→~∃xH(d(x)z))∨∃xFa(x). ∴ ~∀x(∀y~H(xy)∧∃zG(xz))

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3. Prove that the following statement is a theorem valid using a derivation. You may use all rules. (9 marks)

∴ ∀x∃y(~Fx∧Gy)∧∀x(∃yH(a(x)xy)→Fx)→~∀x(~Gx∨∀yH(yxa))

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4. Prove that the following argument is valid using a derivation. You may use all rules. (9 marks)

∃x(~Bx∧Ax)→∃x∀yG(b(x)y). ∴ ∀x∃y~(Ax→By)→∃xG(xa(x))∧∃yG(yy)

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Total = 100Marks

**AID SHEET: Derivation Rules (2 pages)**

**Derivation Types:**

**Direct Derivation (DD or dd)**

**Conditional Derivation (CD or cd)**

**Indirect Derivation (ID or id)**

**Universal Derivation (UD or ud)** Restriction: the instantiating term cannot occur  
 unbound in any available line, or in a premise  
 used in an available line.

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**10 Basic Rules for Sentential Operators:**

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| **Modus Ponens (MP or mp)**  (φ → ψ)  φ  ⎯⎯⎯⎯  ψ | | | | **Modus Tollens (MT or mt)**  (φ → ψ)  ~ψ  ⎯⎯⎯⎯  ~φ | |
| **Double Negation (DN or dn)** | | | | **Repetition (R or r)** | |
| φ  ⎯⎯⎯  ~~ φ | | ~~φ  ⎯⎯⎯  φ | | φ  ⎯⎯⎯  φ | |
| **Simplification (S/SL/SR or s/sl/sr)** | | | | **Adjunction (ADJ or adj)** | |
| φ ∧ ψ  ⎯⎯⎯⎯  φ | | | φ ∧ ψ  ⎯⎯⎯⎯  ψ | φ  ψ  ⎯⎯⎯⎯  φ ∧ ψ | |
| **Addition (ADD or add)** | | | | **Modus Tollendo Ponens (MTP or mtp)** | |
| φ  ⎯⎯⎯⎯  φ ∨ ψ | ψ  ⎯⎯⎯⎯  φ ∨ ψ | | | φ ∨ ψ  ~ φ  ⎯⎯⎯⎯  ψ | φ ∨ ψ  ~ ψ  ⎯⎯⎯⎯  φ |
| **Biconditional-Conditional (BC or bc)** | | | | **Conditional-Biconditional (CB or cb)** | |
| φ ↔ ψ  ⎯⎯⎯⎯  φ → ψ | | | φ ↔ ψ  ⎯⎯⎯⎯  ψ → φ | φ → ψ  ψ → φ  ⎯⎯⎯⎯  φ ↔ ψ | |

**Derived Rules for Sentential Operators:**

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| **Negation of Conditional**  **(NC or nc)** | ~(φ → ψ)  ⎯⎯⎯⎯  φ ∧ ~ψ | | | φ ∧ ~ψ  ⎯⎯⎯⎯  ~(φ → ψ) | | |
| **Conditional as Disjunction**  **(CDJ or cdj)** | φ → ψ  ⎯⎯⎯⎯  ~φ ∨ ψ | ~φ ∨ ψ  ⎯⎯⎯⎯  φ → ψ | | ~φ → ψ  ⎯⎯⎯⎯  φ ∨ ψ | φ ∨ ψ  ⎯⎯⎯⎯  ~φ → ψ | |
| **Separation of Cases**  **(SC or sc)** | φ ∨ ψ  φ → χ  ψ → χ  ⎯⎯⎯⎯  χ | | | φ → χ  ~φ → χ  ⎯⎯⎯⎯  χ | | |
| **De Morgan’s**  **(DM or dm)** | ~ (φ ∨ ψ)  ⎯⎯⎯⎯  ~φ ∧ ~ψ | | ~ (~φ ∨ ~ψ)  ⎯⎯⎯⎯⎯  φ ∧ ψ | ~ (φ ∧ ψ)  ⎯⎯⎯⎯  ~φ ∨ ~ψ | | ~ (~φ ∧ ~ψ)  ⎯⎯⎯⎯⎯  φ ∨ ψ |
| ~φ ∧ ~ψ  ⎯⎯⎯⎯  ~ (φ ∨ ψ) | | φ ∧ ψ  ⎯⎯⎯⎯⎯  ~ (~φ ∨ ~ψ) | ~φ ∨ ~ψ  ⎯⎯⎯⎯  ~ (φ ∧ ψ) | | φ ∨ ψ  ⎯⎯⎯⎯⎯  ~ (~φ ∧ ~ψ) |
| **Negation of Biconditional**  **(NB or nb)** | ~ (φ ↔ ψ)  ⎯⎯⎯⎯  φ ↔ ~ ψ | | | φ ↔ ~ψ  ⎯⎯⎯⎯  ~ (φ ↔ ψ) | | |

**Derivation Rules for Predicate Logic:**

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| **Basic Rule:**  **Existential**  **Generalization**  **(EG or eg)** | **Basic Rule:**  **Universal Instantiation (UI or ui)** | **Basic Rule:**  **Existential Instantiation  (EI or ei)** | **Derived Rule:**  **Quantifier Negation (QN or qn)** | | **Derived Rule:**  **Alphabetic Variance  (AV or av)** | |
| φζ  \_\_\_\_\_  ∃αφα  Restriction: α does not occur as a free or bound variable in φζ. | ∀αφα  \_\_\_\_\_  φζ  Restriction: ζ does not occur as a bound variable in φα | **∃**αφα  \_\_\_\_\_  φζ  Restriction: ζ does not occur in any previous line or premise. | ∀α~φ  \_\_\_\_\_ ~∃αφ | ~∀αφ  \_\_\_\_\_  ∃α~φ | ∀αφα  \_\_\_\_\_  ∀βφβ | ∃αφα  \_\_\_\_\_  ∃βφβ |
| ~∃αφ  \_\_\_\_\_  ∀~αφ | ∃α~φ  \_\_\_\_\_  ~∀αφ | Restriction: β does not occur as a bound variable in φα | |

Total Pages (14)